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Estimating Real GDP Growth for Lebanon

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ABSTRACT

This paper presents an econometric framework to estimate quarterly GDP growth for Lebanon based on a bottom up approach from the demand side. The model relies on a data set of 68 quarterly observations from 1993 to 2009 for ten endogenous variables and two exogenous variables selected on the basis of their economic and statistical significance. Quarterly GDP figures are obtained from the annual series using the Chow-Lin disaggregation method. The model derived is a Vector Autoregressive Model with Exogenous Variables (VARX), a variant of the Vector Autoregressive Model (VAR) that takes into account both exogenous and endogenous variables. Our empirical results show robust correlation between the estimate and actual quarterly GDP figures, indicating the ability of the model to provide a high level of accuracy in estimating real GDP growth.

JEL Classification Numbers: C51, C32, C13, C82, O53

Keywords: Granger causality, VAR, VARX, impulse response function, and GDP estimate.

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I- Introduction

Macroeconomic statistics have historically been poor in Lebanon. The lacks of public investments in institutions, the low public wages that make it difficult to attract highly qualified people, and the lack of understanding, on the political front, of the importance of having adequate national accounts and up to date information for decision making, have left the country with weak public institutions. Even after the end of the civil war and the starting of the reconstruction phase, Lebanon did not build its statistical capabilities and the country was left without serious national accounts until the early years of the past decade (2000-2009).

Up until now, Lebanon was not able to put in place a real independent institution responsible for macroeconomic statistics. The national accounts are not being performed at the Central Administration for Statistics. It is a unit at the prime minister's office that is responsible of issuing the national accounts. Consequently, the independence of this unit and its ability to perform its duties without any political interference are questioned.

Even in the private sector, there is no research institution that issues working papers and real scientific surveys on the economic activity. The private sector is not feeling the urgency of investing in such non-for-profit organizations. In spite of the importance of the subject, however, estimating macroeconomic variables was quite avoided in Lebanon. The poor data gathering and mediocre measurement capabilities make it a daunting task for anyone to carry it out.

The result is that wrong decisions may be taken on both the public and the private sectors fronts for lack of adequate and up to date data. For example, the official real growth rate of the economy for 2009 is not out yet, and at best we have guesstimates from the central bank and some international institutions.

Hence the frequency and scarcity of available data along with lags in releases of macroeconomic indicators constitute the main challenges to estimating GDP. Lebanon's national accounts are compiled on a yearly basis with actual GDP published with a nine-month to two-year lag. In addition, the unavailability of key macroeconomic data prior to 1990s, due mainly to the effects of war on data gathering, results in several breaks in various series of economic variables.

Going back to before the civil war, the evolution of the Lebanese economy has followed a cycle of development that shaped Lebanon into an open, liberal and service-oriented one. The country has had a dynamic growth in the years leading up to the Civil War with GDP rising 6% per year from 1965 to 1975. A year prior to war, GDP stood at $3.5 billion on the back of an increasing productivity, a strong Lebanese pound and flourishing banking and tourism sectors.

In the years that followed the Civil war, Lebanon has entered an era where reliable statistics on the state of the economy were usually absent. Lebanese economists were sometimes able to compile a few indicators, but the numbers were often based on incomplete data.

In the aftermath of the war, the advent of the Hariri Government in October 1992, led to the restoration of the country's infrastructure and lifted up the economic activity on all levels. Solidere managed the reconstruction of Beirut's central business district and the stock market reopened in January 1996. GDP expanded from $1.3 billion in 1990 to $16.7 billion in 2000.

The private sector was the engine for economic recovery after the war, and till today, it still stands as the principle pillar for economic growth sustainability through services (mainly tourism, real estate and banking) sectors that combined, represent 70% of GDP. The economy today, however, is heavily indebted with gross public debt totaling $54.4 billion and accounting for 139% of GDP. Political instability poses as well a severe hurdle for economic growth.

In this context, estimating economic growth has a crucial purpose to fill within the large field of policy making and it represents one of the basic problems of statistical analysis upon which authorities rely to set the right policies. In this paper, we try to present a general model to obtain quarterly real GDP growth estimates. Current GDP levels may provide only insufficient information on future macroeconomic developments. GDP estimates that link near-future growth to current development, can bridge this gap.

During the conduct of our work, we were faced with two major challenges: the first one was related to available series of data and the second one was to try to come up with an accurate and scientific model to estimate real GDP growth rate. For the first challenge we had annual GDP series going back to just 1993, which does not constitute enough data to build the model. So we had to transform the annual GDP numbers into quarterly data to increase the size of our sample and to get real growth rates estimates on a quarterly basis, which does not exist for Lebanon. For the second challenge, we tried to build our model around a set of variables using a vector autoregressive (VAR) approach. For variables other than GDP, we had monthly series going back to 1993.

The rest of the paper is organized as follows. Section II and III provide an overview of the different GDP estimation methods applied worldwide with a focus on the case of Lebanon. Section IV and V, present a potential Vector Autoregressive Model (VAR) for GDP estimation that was developed in this paper and address its limitation. Section VI provides a variant of the VAR, a Vector Autoregressive Model with Exogenous variables (VARX), which offers a better precision in estimating real GDP and overpass the limitations encountered with the VAR. Section VII describe the impulse response of real GDP to shocks on various variables. The appendix lay out the complete details of the specifications used in the empirical application.

II- Literature Review

Disaggregation methods to obtain quarterly GDP from annual GDP have been extensively considered in econometric and statistical literature. The many proposed solutions have been reached using one of the following two approaches:

The first one being a method which estimates the disaggregated series (e.g. quarterly GDP) using information derived only from past and current values of the aggregated series itself (e.g. annual GDP_t; annual GDP_{t-1}). This approach does not involve the use of parameters as no additional exogenous variables are considered. We distinguish here between non-model based methods, Polynomial (1988), Lisman & Sandee (1964) and model based methods, Stram & Wei (1986). The former relies on purely numerical disaggregation technique. An example would be to divide the annual data into a quarterly figure. This approach is known as linear interpolation and it is mostly used for stocks disaggregation. Another example of non-model based method, is the Polynomial method (1988) which converts the annual series to quarterly figures by fitting a polynomial to each successive set of two points (e.g. GDP_{2000} and $GDP_{Q1/2001}$) to derive a smooth path for the unobserved series. The model-based methods use an ARIMA process.

The second methodology, with more interest for our purposes, consists of a method that presents a disaggregation scheme (e.g. for quarterly GDP disaggregation) based on information which comes from the aggregated series itself and also from other exogenous series, called related series (e.g. consumption of durable goods, volume of exports). The related series are assumed to be known at the same disaggregation level as the considered disaggregated series (i.e. quarterly GDP in our case). This approach is model-based and uses correlated time series to provide estimates of the disaggregated series based on the parameters ($\hat{\beta} and\ \hat{u}$) of the aggregated series. The method was adopted by Friedman (1962), and by Chow and Lin (1971). Variants of it were proposed by Bournay and Laroque (1979), Fernandez (1981) and Litterman (1983).

Andreas Kladroba (2005) carried out a study to compare different methods for a simulated series from an ARIMA (1, 1, 1) and showed that Chow-Lin is the most accurate for this relative simulated series.

Regarding GDP estimation, two methods were developed. The first one consists of developing indicators relying on a non-model based methodology. This approach was adopted by Carriero and Marcellino (2010) and by the Conference Board that constructs the Composite Coincident Indicator (CCI) for the United States as a simple weighted average of selected standardized single indicators.

The second type, adopted in this paper, consists of developing a model-based quarterly estimate of GDP derived through bottom-up approach based on actual values of the quarterly components which serve as proxies for selected indicators. There exists the non-parametric method; an example to cite is the Neural Network with an economic application; and the parametric models. The latter include the Vector Autoregressive model (VAR) which is adopted in this paper, the Error Correction Model (VECM), and the Markov Switching Models (MSVAR) which is an updated version of the VAR. Stochastic models were also built and used such as the Dynamic Stochastic General Equilibrium (DSGE) model that provides a continuous path for the estimated variable (e.g. provides a graphical trend for GDP over the whole quarter instead of a stock value).

The European Central Bank uses log-linear approximations to deal with real-time data set and estimates GDP for the euro area using a VAR model. Braun (1991) uses a Bayesian vector autoregression (BVAR) to smooth out US production and fill in the missing data for a given quarter. The Bureau of Economic Analysis (BEA) estimates GDP on an annual and on a quarterly basis. The first estimate for a certain quarter, namely the "Advance" estimate is done by extrapolation based on the monthly trends as incomplete data account for about 30% of the advance GDP estimate.

III- GDP Disaggregation in the Case of Lebanon

We estimate GDP for Lebanon from the demand side, relying on indicators and predictors in different sectors of the economy as proxies to consumption, investment, and trade. The model proposed is the first GDP estimate model developed for Lebanon.

Other estimates are only indicators for the trend in economic activity. The Banque du Liban (BDL) and the International Institute of Finance (IIF) have designed indices to measure the evolution of the country's economic activity. The former was developed in 1993 and is called "Coincident Indicator Index". It relies on eight indicators: electricity production, imports of petroleum, passengers flows, cement derivatives (all in volume terms), imports and exports, cleared checks, and broad money. The latter follow the BDL's approach, and includes five additional indicators: real growth in credit to the private sector (to substitute for growth in deposits), growth in tourist arrivals (to substitute for passengers arrivals), real growth in government revenues excluding grants, real growth in government consumption (current expenditure minus transfers minus interest payments), and real growth in imports of machinery and equipment. Electricity production has been excluded because, according to IIF, it does not accurately reflect consumption in Lebanon, since a significant portion of electricity is derived from private generators. We second the argument and therefore exclude it from our model.

We used the Chow and Lin approach to disaggregate annual GDP into quarterly levels. The Chow and Lin solution (1971) has been intensively used in National Statistical Institutes. The reason lies in the practicality of its procedure and in the natural and coherent solution that the model provides to the interpolation problem.

Assume that GDP series are available annually over n years. Let y_q be a $(4n \times 1)$ vector of quarterly GDP figures to be estimated and let X be a $(4n \times k)$ matrix of k exogenous quarterly GDP-related variables.

Then quarterly y_q can be predicted using a multiple linear regression of the form:

$$y_{q=}X_q \, \beta + \hat{u}_q \qquad (1)$$

Where $\hat{u}_q$ is a $(4n \times 1)$ random vector. By assumption, Chow and Lin have considered the error term $\hat{u}_q$ to follow a stationary first order autoregression $u_{q,t} = \rho \, u_{t-1} + e_t$ (where e_t is white noise and $|\sigma| < 1$) with having zero mean and covariance matrix

$$V=\frac{\sigma^2}{1-\rho^2}\begin{pmatrix} 1 & \rho & \rho^3 & & \rho^{4n-3} & \rho^{4n-2} & \rho^{4n-1} \\ \rho & 1 & \rho & \cdots & \rho^{4n-4} & \rho^{4n-3} & \rho^{4n-2} \\ \rho^2 & \rho & 1 & & \rho^{4n-5} & \rho^{4n-4} & \rho^{4n-3} \\ & \vdots & & \ddots & & \vdots & \\ \rho^{4n-3} & \rho^{4n-4} & \rho^{4n-5} & & 1 & \rho & \rho^2 \\ \rho^{4n-2} & \rho^{4n-3} & \rho^{4n-4} & \cdots & \rho & 1 & \rho \\ \rho^{4n-1} & \rho^{4n-2} & \rho^{4n-3} & & \rho^2 & \rho & 1 \end{pmatrix}$$

Now let C be a $(n \times 4n)$ matrix that converts *4n* quarterly observations into n annual observations. The matrix is defined as:

$$C = \begin{bmatrix} 11110000 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1111 \end{bmatrix} \qquad (2)$$

Using subscript a to denote annual figures, equation *(1)* can be converted to a regression of annual aggregates:

$$y_a = C \, y_q = CX_q \, \beta + C\hat{u}_q = X_a \, \beta + u_a \qquad (3)$$

To obtain the Chow and Lin (1971) equation that disaggregates *n* annual GDP estimates to $4n$ quarterly estimates, we apply the Generalized Least Square (GLS) method to equation *(3).* This yield:

$$\hat{\beta}_q = (X_a^{'} \, V_a^{-1} \, X_a)^{-1} \, (X_a^{'} \, V_a^{-1} \, y_a) \qquad (4)$$

And

$$\hat{u}_a = y_a - X_a \, \hat{\beta} \qquad (5)$$ *(showing no serial correlation)*

Where $V_a = (CVC^{'})$ (6)

Using the previous four equalities, the Chow Lin best linear unbiased estimate (BLUE) of quarterly GDP y_q is derived from:

$$\hat{y}_q = X_q\,\hat{\beta} + VC^{'}(CVC^{'})^{-1}\,\hat{u}_a$$

We obtain an equation of degrees 7 in the unknown ρ with the form:

$$\frac{\rho^7 + 2\rho^6 + 3\rho^5 + 4\rho^4 + 3\rho^3 + 2\rho^2 + \rho}{2\rho^3 + 4\rho^2 + 6\rho + 4} = \widehat{\rho_a}$$

Where $\widehat{\rho_a}$ is the autocorrelation factor of the annual residual u_a

The estimates of quarterly GDP y_q in Chow and Lin (1971) model are based on exogenous quarterly variables X_q and estimated $\hat{\beta}$ from annual totals for the following five variables: Claims on Private Sector, Petroleum Imports, Non-residents Spending by Credit Cards, Number of Arrivals at the Beirut International Airport and Consumer Price Index. The annual residuals $\hat{u}_a$ are allocated to the four quarters of the year such that the annual sum of the disaggregated quarterly values y_q equal to y_a

IV- Choice and Characteristics of Variables

Turning back to the model developed in this paper, we tried through the choice of our variables to have proxies for public and private consumption and investment, and the current account position. Therefore, we started with the following variables:

"Number of Tourists' Arrival at Beirut International Airport (BIA)": These tourists have a huge impact on the exports of services and on local consumption. However, we seconded this statistic by two other more general ones that are "Total Arrivals" and "International Air Passenger Flows at BIA". These variables will include the Lebanese expatriates who visit Lebanon each year and contribute substantially to its economy. These expats are also more resilient than normal tourists to political shocks. The latter variable does also comprise the departures from BIA.

"Cement Production": This variable may be used as a proxy for investment in real estate and infrastructure. Hence it gives an idea about both private and public sectors investments in the real estate sector. We added another predictor for real estate activity that is "lagged construction permits" as the variable's impact on the economy is slow due to the time lag between the issuance of the permit and the initiation of construction.

"Claims on Private Sector": This variable is used as a proxy private consumption and private investment as well. Claims on private sector comprise personal and consumption loans contracted by individuals for their purchases of cars, furniture, daily consumption, etc. The variable also includes loans allocated to businesses for their expansion purposes and thus will in this regard represent investment of the private sector.

"Petroleum Imports": This variable will give an idea about the consumption level and more generally about the economic activity as an increase in consumption and economic activity will lead to an increase in the petroleum imports.

"Government Spending": The considered variable comprises primary spending, thus public debt service has been excluded. Primary government spending comprises both current spending that represents public consumption and capital spending that represents public investment.

"Imports of Goods (Excluding Petroleum)" and "exports of goods": These variables serve as proxies to measure the external sector contribution to the economy, in addition to the importance of imports as a proxy to consumption as Lebanon imports more than 80% of its consumption goods.

"Broad Money M3": This variable represents the impact of monetary policy on the liquidity available in the market, in addition to the inflow of capital and remittances. So an increase in M3 will most probably have a positive impact on GDP.

"Consumer Price Index": Besides using the CPI to deflate our nominal variables, we also included the CPI in our model since high inflation may negatively impact growth, and Lebanon went through high inflation rates in the 90s. We rebased the CPI to the year 1993 to have a complete time series.

"Cleared Checks": We thought that the amount or the number of cleared checks will be useful to represent the general economic activity as the increase in the number and amount of cleared checks will probably lead to an improvement in the economic activity. This could be used mainly as another proxy for consumption.

"Non-residents spending by credit cards": This variable is used as a proxy for the additional exports of goods and services that are not being taken into account by the exports of goods that are taken from customs and BDL.

Our sources of data are mainly the websites of the Ministry of Finance and Banque du Liban (BDL).

The figures extracted from the database are available at monthly frequency and so to adjust to quarterly series, we adopted two different approaches depending on the nature of the indicators: if the series under construction is that of a flow variable measured over an interval of time, we resort to adding up the three months figures of each quarter to obtain the quarterly value. If the series considered relates to a stock variable measured at one specific time, we consider the average value of the three monthly figures of each quarter as the quarterly value. The approach adopted in the latter case, allows the derived quarterly figure to capture the changes registered over the course of the considered quarter.

Before proceeding with the model, we seasonally adjust the real variables, computed by deflating the nominal variables with the CPI, to remove the noise effects that hide the underlying trend in real short term changes. The seasonally adjusted series are derived using the Census X-12 algorithm used by the US Bureau of the Census. The procedure consists of decomposing a time series into three components among which the seasonal component and the irregular component. We use the multiplicative seasonal adjustment decomposition model to filter the influence of seasonality, as the magnitude of the seasonal fluctuations vary with the level of the series (e.g. number of tourists arrival) which are of positive values.

We apply the Granger Causality test (1969) to determine the endogenous variables for the model. The test shows whether lagged information on a variable Y provides any statistically significant information about a variable X in the presence of lagged X. If not, then "Y does not Granger-cause X". And so X is said to be exogenous. If "Y Granger cause X" and "X granger cause Y" then X and Y are said to be endogenous.

The application of the test reduced the number of the endogenous variables of the model to ten out of fourteen initial variables: GDP, Import Petroleum, Claims on Private Sector, Cement Production, Total Imports (excluding Petroleum), Arrivals at the Beirut International Airport, Government Spending, Total Exports, CPI, and Non-resident Spending by Credit Cards.

The remaining four variables are exogenous to the model and include: Money Supply (M3), Lag of construction permits, cleared checks, and Number of Tourists.

V- The Vector Autoregressive Model (VAR)

The Vector Autoregression (VAR) allows for the forecast of time series and the analysis of dynamic impact of random disturbances on the system of variables. The VAR approach considers every endogenous variable as a function of the lagged values of all the endogenous variables in the system.

We define Y_t as:

$$Y_t = (Y_{1t}, Y_{2t}, \dots, Y_{10t})'$$

$$Where\ Y_{it}, for\ i = 1 \dots 10\ are\ the\ endogenous\ variables$$

The VAR (vector autoregressive model) is derived as follow:

$$Y_t = c + \Phi_1 Y_{t-1} + \Phi_2 Y_{t-2} + \cdots + \Phi_p Y_{t-p} + \varepsilon_t$$

$$Where\ c\ denotes\ a\ (10\ x\ 1)\ vector\ of\ constants\ (c_1, c_2, \dots, c_{10})'$$

$$\Phi_j = \begin{pmatrix} \Phi_{11}{}^{(j)} & \cdots & \Phi_{110}{}^{(j)} \\ \vdots & \ddots & \vdots \\ \Phi_{91}{}^{(j)} & \cdots & \Phi_{910}{}^{(j)} \end{pmatrix} \text{for } j = 1,2,\dots,p$$

$$And\ \varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, \dots, \varepsilon_{10t})'\ with$$

$$E(\varepsilon_t) = 0_{10\times10}\ \ and$$

$$E(\varepsilon_t \varepsilon_\tau{}') = \begin{cases} \Omega\ for\ t = \tau \\ 0_{10\times10}\ otherwise \end{cases}\quad Where\ \Omega\ is\ a\ (10 \times 10) symmetric\ positive\ definite\ matrix$$

To determine the lag order of the VAR model, we use a general simple model selection approach that yields models with likelihood functions and a finite number of estimated parameters. For that purpose, we use the Akaike Information Criterion (Akaike, 1974) that presents a simple model comparison criterion that uses a penalty term to penalize the log maximum likelihood for lack of closeness. The application of Akaike Criterion yields the following:

Table 1: *VAR Lag Order Selection Criteria*

VAR Lag Order Selection Criteria

Lag	*LogL*	*AIC*	*SC*	*HQ*
0	-1303.796	45.2812	46.408	45.72106
1	-1198.058	42.23925	45.648465	42.89904
2	-1176.637	42.05549	44.30909	42.9352
3	-1143.457	41.47313	44.29013	42.57278
4	**-1120.589**	**41.24032***	**43.92945***	**42.55989***
5	-1109.079	41.3925	45.3363	42.932

** indicates the selected lag order*

AIC for Akaike information criterion

SC for Schwarz information criterion

HQ for Hannan-Quinn information criterion

The choice of the appropriate length of a lag distribution happens at the lag order that yields the lowest value of the Akaike Criterion. From the table the proper lag order for the considered VAR reads 4.

Alternative general criteria exist for model selection. The most common are the Hannan-Quinn Criterion (1979) and the Schwartz Criterion (1978). For the considered VAR model, the latter two methods yield the same outcome as the Akaike Criterion, strengthening the choice of the adopted model selection. This is expected to a certain extent as a lag order of 4 means that the effect of a variable change in a specific quarter will impact GDP for the whole year.

We proceed by estimating VAR of lag order 4:

$$Y_t = c + \Phi_1 Y_{t-1} + \Phi_2 Y_{t-2} + \Phi_3 Y_{t-3} + \Phi_4 Y_{t-4} + \varepsilon_t$$

$With\ \varepsilon_t \rightarrow iid\ N(0, \Omega)$

We use the maximum likelihood estimator to estimate the model and derive the parameters.

A crucial condition for the VAR model to be valid and consistent requires the covariance to be stationary (i.e. time invariant) in order to avoid the formation of explosive roots. We test for the stationarity of VAR using the lag operator $LY_t = Y_{t-1}$

This translates into:

$$Y_t = I_{10} \times Y_t = c + \Phi_1 L Y_t + \Phi_2 L^2 Y_t + \Phi_3 L^3 Y_t + \Phi_4 L^4 Y_t + \varepsilon_t$$

$$[I_{10} - \Phi_1 L - \Phi_2 L^2 - \Phi_3 L^3 - \Phi_4 L^4] Y_t = c + \varepsilon_t$$

Equivalent to:

$$\Phi(L) Y_t = c + \varepsilon_t$$

$Where\ \Phi(L)\ indicates\ a\ (10\ x10)\ matrix\ of\ polynomial\ in\ the\ lag\ operator\ L:$

$$\Phi(L) = I_{10} - \Phi_1 L - \Phi_2 L^2 - \Phi_3 L^3 - \Phi_4 L^4$$

We replace the lag operator L with a scalar z. So the stationarity of VAR requires the roots of

$$|I_{10} - \Phi_1 z - \Phi_2 z^2 - \Phi_3 z^3 - \Phi_4 z^4|$$

to lie outside the unit circle (have modulus greater than one) or equivalently, if the eigenvalues of the companion matrix:

$$F = \begin{pmatrix} \Phi_1 & \Phi_2 & \Phi_3 & \Phi_4 \\ I_{10} & 0 & 0 & 0 \\ 0 & I_{10} & 0 & 0 \\ 0 & 0 & I_{10} & 0 \end{pmatrix}$$

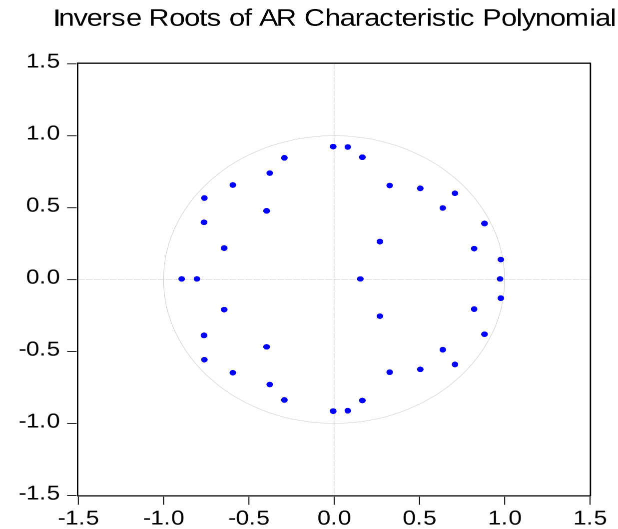
Which are those numbers λ that satisfy:

$$|F - \lambda I_{10}| = 0$$

Are less than one.

In our model, the eigenvalues are derived in Eviews. Graphically, the result confirms the stationarity of VAR[1]:

Figure 1: *Inverse Roots of AR*



Therefore, our VAR seems to be stationary (covariance stationary) with its first and second moments time invariants[2].

VAR model holds strongly and passes the stationarity and normality (Jacque-Bera) tests. However, the relatively small adjusted R^2 (0.83) reflects the weak ability of the model to predict a trend. In fact, the backward testing, that provides a comparison between the actual values and the estimated values derived from the model over previous periods, shows a significant difference between the two values considered for the same period[3]. Therefore, in order to improve the predictability of the model we include the exogenous variables discussed earlier, as their inclusion might bring in additional information and contribute for a better trend predictability. To achieve that we build a more sophisticated model which is a generalization of VAR, called VARX (i.e. VAR with exogenous variables).

[1] The eigenvalues are graphically represented by the inverse roots and their numerical values are computed in Appendix A-2

[2] For a detailed approach on how to get the eigenvalues refer to Appendix A-1

[3] For further information on VAR refer to Appendix B

VI- Vector Autoregressive with Exogenous Variables (VARX)

We consider Y_t previously defined as:

$$Y_t = (Y_{1t}, Y_{2t}, \dots, Y_{10t})'$$

Where $Y_{it}{}'$ *are the endogenous variables*

Recalling that the Granger causality test conducted in section IV showed that four out of the fourteen variables under consideration are exogenous, we let $(X_{1t})_t; \dots; (X_{5t})_t$ be the time series of the considered 5 exogenous variables.

VARX is defined as:

$$Y_t = c + \Phi_1 Y_{t-1} + \cdots + \Phi_4 Y_{t-4} + \omega X_t + u_t$$

W*here c denotes a* (10×1) *vector of constants*

$(\Phi_j)_{j=1,\dots,p}$ *are* (10×10) *matrices of autoregressive coefficients*

$$X_t = (X_{t1}, \dots, X_{t5})'$$

$$\omega = \begin{pmatrix} \beta_1^1 & \cdots & \beta_1^5 \\ \vdots & \ddots & \vdots \\ \beta_{10}^1 & \cdots & \beta_{10}^5 \end{pmatrix} \text{ where } (\beta^i)_{i=1,\dots,5} \text{ are coefficients of exogenous variables in each equation}$$

And u_t *is a* (10×1) *vector generalization of white noise with:*

$$E(u_t) = 0$$

$$And\ E(u_t u_\tau') = \begin{cases} \Omega\ for\ t = \tau \\ 0\ otherwise \end{cases} \quad with\ \Omega\ an\ (10 \times 10)\ symmetric\ positive\ definite\ matrix$$

VARX passes the normality and stationarity tests.

We estimate the model using the Multivariate least square estimation method.

We proceed by testing for the significance of the exogenous variables$(X_{1t})_t; \dots; (X_{5t})_t$ using the t-test. Only two out of the four initial exogenous variables pass the test: money Supply (M3) and lag of construction permits.

VARX is written as:

$$Y_t = c + \Phi_1 Y_{t-1} + \cdots + \Phi_4 Y_{t-4} + \omega X_t + u_t$$

Table 2: *Vector Autoregressive Model with Exogenous variables (VARX)*

Vector Autoregressive Estimates

	GDP	IP	L(C)	L(CP)	AR	L(GS)	L(TI)	TE	CPI	NRS
GDP(-1)	-0.158157	1.77E-05	-7.64E-11	-0.005453	-6371.397	0.001393	-1.49E-10	-2.17E-06	2.30E-11	-6.90E-05
GDP(-2)	-0.398408	1.49E-05	9.78E-12	0.093154	1115.228	0.012842	1.28E-11	-2.63E-06	9.65E-10	-5.16E-05
GDP(-3)	-0.283008	1.59E-05	-1.89E-10	-0.027843	4013.01	0.073112	-4.10E-10	-4.66E-06	-2.50E-09	-9.50E-05
GDP(-4)	0.373177	-2.42E-06	-5.42E-11	0.111198	-9963.51	-0.066573	-1.15E-10	6.94E-07	-5.04E-10	-7.24E-06
IP(-1)	7.24E-07	-1.33E-11	6.60E-17	-1.46E-07	-0.026973	4.50E-08	1.35E-16	2.99E-12	3.88E-16	8.59E-11
IP(-2)	8.18E-07	-2.58E-11	1.31E-16	-5.61E-08	-0.008168	-8.74E-10	2.71E-16	5.99E-12	7.99E-16	1.38E-10
IP(-3)	1.93E-07	-1.64E-11	1.88E-16	9.57E-08	0.007104	-6.88E-08	3.77E-16	5.05E-12	2.26E-15	9.71E-11
IP(-4)	3.17E-07	-1.01E-12	-1.92E-17	-1.36E-07	-0.00171	5.58E-08	-1.21E-17	8.45E-13	4.59E-16	1.52E-11
L(C(-1))	-3.125236	-0.000219	4.39E-09	-0.490483	75762.5	-0.258296	9.76E-09	0.000104	8.48E-08	0.001522
L(C(-2))	2.951557	0.000557	-3.61E-09	0.846472	-139089.1	0.564217	-7.36E-09	-0.000112	-4.48E-08	-0.002026
L(C(-3))	-0.122233	-0.000679	3.31E-09	1.643978	59460.6	-0.278702	5.71E-09	6.49E-05	1.33E-09	0.002315
L(C(-4))	-3.571442	0.000335	-1.73E-09	-1.681804	82001.08	0.166738	-2.94E-09	-3.20E-05	2.34E-09	-0.001399
L(CP(-1))	-0.09504	2.53E-05	4.76E-11	0.404298	25597.46	-0.037485	1.79E-10	4.31E-06	4.79E-09	-1.12E-05
L(CP(-2))	0.399649	-3.82E-05	-4.61E-11	0.255901	-10135.27	0.045848	-1.70E-10	-1.57E-06	-5.75E-09	7.48E-05
L(CP(-3))	0.161266	-2.22E-05	2.79E-10	0.307033	15171.69	-0.031441	5.59E-10	7.45E-06	4.07E-09	0.000114
L(CP(-4))	-1.196725	5.15E-05	-2.13E-10	-0.250255	-2127.578	0.146523	-4.13E-10	-1.23E-05	-1.56E-09	-0.000225
AR(-1)	7.67E-07	-5.93E-12	2.25E-16	-5.50E-07	0.154127	-1.25E-07	5.63E-16	9.00E-12	5.45E-15	8.08E-11
AR(-2)	-1.22E-06	5.25E-11	-2.26E-16	1.43E-07	0.099503	-5.55E-07	-3.29E-16	-4.92E-12	2.02E-15	-2.50E-10
AR(-3)	2.06E-06	-1.14E-10	7.48E-17	-1.26E-06	0.056046	-1.00E-09	3.46E-18	4.91E-12	-1.17E-14	3.63E-10
AR(-4)	-1.72E-06	1.72E-12	2.46E-16	1.39E-07	0.135737	-6.76E-07	5.22E-16	9.61E-12	6.39E-15	7.29E-12
L(GS(-1))	0.291391	4.19E-06	-1.18E-11	0.050389	13227.95	0.022697	-2.34E-11	1.73E-06	3.24E-10	-4.71E-06
L(GS(-2))	0.313212	-1.68E-05	1.44E-11	0.300915	9792.857	-0.027367	3.22E-12	1.54E-06	-8.62E-10	4.26E-05
L(GS(-3))	0.515204	-1.83E-05	-4.41E-12	0.149105	-9241.982	-0.02034	-1.08E-11	2.91E-06	-7.76E-10	6.66E-05
L(GS(-4))	-0.40663	-1.06E-05	1.24E-10	-0.066242	-7643.163	-0.010927	2.47E-10	1.43E-06	1.01E-09	5.50E-05
L(TI(-1))	0.13861	-1.84E-05	1.94E-12	0.120685	-761.4418	0.119553	-3.72E-11	2.41E-06	-1.86E-09	6.30E-05
L(TI(-2))	0.071641	-1.80E-05	4.22E-11	0.077129	3910.502	-0.065102	9.72E-11	1.38E-06	-5.48E-10	3.71E-05
L(TI(-3))	0.205535	9.85E-06	-1.19E-10	0.01608	-6336.961	0.00177	-1.90E-10	-1.03E-06	1.15E-10	-3.43E-05
L(TI(-4))	-0.057933	2.03E-07	3.72E-11	-0.07973	-6971.123	0.045937	2.62E-11	-2.99E-07	-5.20E-10	4.26E-06
TE(-1)	1.66E-06	-2.30E-12	-4.20E-16	6.02E-08	0.03981	-1.48E-07	-9.59E-16	-7.01E-12	-9.00E-15	-1.02E-10
TE(-2)	2.72E-06	-1.74E-10	4.82E-16	-3.78E-07	0.026443	-1.13E-07	9.28E-16	3.70E-11	7.97E-16	7.50E-10
TE(-3)	1.75E-06	-1.63E-11	5.30E-16	3.17E-07	-0.056502	-4.72E-07	1.27E-15	2.01E-11	1.22E-14	2.88E-10
TE(-4)	7.57E-07	-7.87E-11	9.72E-16	-3.67E-07	-0.009068	3.47E-07	2.04E-15	1.78E-11	1.10E-14	5.67E-10
CPI(-1)	0.008631	1.25E-07	7.09E-13	0.002713	179.8713	-0.000323	2.60E-12	6.34E-08	4.31E-11	-2.26E-07
CPI(-2)	-0.022179	1.89E-07	4.40E-12	-0.000425	482.6875	-0.001921	7.85E-12	-1.15E-07	2.98E-11	-8.30E-07
CPI(-3)	0.004989	3.36E-07	-4.79E-12	-0.003002	489.4756	0.001155	-9.46E-12	-7.19E-08	-5.04E-11	-1.71E-06
CPI(-4)	-0.003752	-1.22E-06	2.90E-12	0.008752	417.1511	-0.004178	3.92E-12	1.92E-07	-1.63E-11	3.67E-06
NRS(-1)	-1.25E-07	3.07E-13	-1.04E-17	-4.12E-08	-0.002733	-9.43E-09	-2.71E-17	-9.35E-13	-3.76E-16	-1.14E-11
NRS(-2)	-1.00E-07	7.97E-12	-1.14E-17	-6.94E-08	-0.00059	2.67E-08	-2.66E-17	-1.23E-12	1.66E-16	-2.71E-11
NRS(-3)	-1.88E-07	5.36E-12	8.69E-18	6.30E-08	0.004028	1.99E-09	2.77E-17	-1.71E-13	6.88E-16	-1.63E-11
NRS(-4)	1.85E-07	-3.89E-12	-3.02E-17	1.28E-08	-0.004903	3.42E-09	-6.14E-17	-1.97E-13	-8.06E-16	3.27E-12
M3	1.51E-05	1.91E-10	1.06E-15	-2.81E-06	-0.725199	-3.02E-07	1.86E-15	2.57E-11	1.90E-14	8.96E-10
LAG(CP,5)	2.08E-14	-6.84E-19	3.76E-24	-4.24E-15	-8.36E-10	-1.52E-15	9.13E-24	2.25E-19	4.50E-23	4.35E-18
R-squared	0.996164	0.945382	0.963344	0.959992	0.999802	0.999239	0.976854	0.956322	0.99924	0.983765
Adj. R-squared	0.97528	9.938883	0.966544	0.742172	0.998726	0.995097	0.973324	0.956322	0.734242	0.973256
Sum sq.	0.101589	4.49E-10	1.64E-20	0.076437	1.13E+08	0.001629	6.04E-20	1.41E-11	3.94E-18	5.52E-09
S.E.	0.106243	7.06E-06	4.27E-11	0.092158	3537.296	0.013455	8.19E-11	1.25E-06	6.61E-10	2.48E-05
F-statistic	47.70012	1.36E+21	4.82E+19	4.407277	929.1223	241.2563	1.28E+19	2.37E+22	8.00E+20	2.74E+22

Where IP= Imports of Petroleum; L(C)= log of Claims on Private Sector; L(CP)= log of Cement Production; AR= Arrivals; L(GS)= log of Government Spending; L(TI)= Total Imports Excluding Petroleum; TE = Total Exports; CPI= Consumer Price NRS= Non-resident Spending by Credit Card; CP= Construction Permits; M3= Money Supply

Some of the coefficients' signs may be unexpected. However, it shows in the impulse response function that for example, any change in claims on the private sector during a specific quarter will lead to an increase of GDP over the upcoming year eventhough LC (-1) and LC(-4) have negative coefficients. The results of the model as a whole were very satisfactory as at shows.

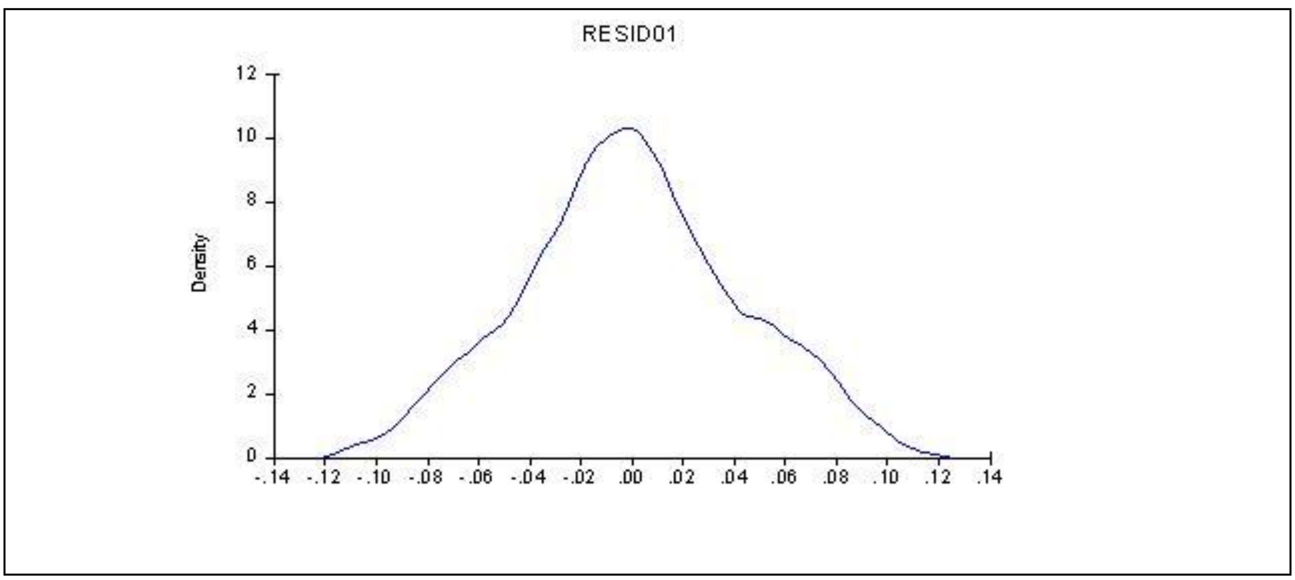
We proceed by testing for the normality of the error terms of VAR using the Jacque-Bera test. The goodness-of-fit test utilizes the information of the sample skewness and sample kurtosis, which are the third and fourth moment respectively and are sensitive to small deviations from normality. The test is conducted in Eviews and accepts normality at 5%.

Table 3: *Normality test – Jacque-Bera*

	RESID01	*RESID02*	*RESID03*	*RESID04*	*RESID05*	*RESID06*	*RESID07*	*RESID08*	*RESID09*	*RESID10*
Jarque-Bera	1.122801	1.059532	9.994863	0.05472	1.53185	0.581345	0.941071	0.694423	0.345286	0.864002
Probability	0.5704	0.5887	0.68	0.973	0.4649	0.7478	0.6247	0.7067	0.8414	0.6492
Result	accept	accept	accept	accept	accept	accept	accept	accept	accept	accept

The normal distribution of residuals can be observed graphically:

Figure 2: *Distribution of residuals*



We have assumed throughout the model residual means of zero. We use the empirical distribution test to prove it. The test, applied to each residual term of a normal series, provides the mean and the standard error of each series and test the significance of each one. Here follows the result of the test applied to the residual term of GDP:

Table 4*: Empirical Distribution Test*

Parameter	Value	Std. Error	z-Statistic	Prob.
MU	1.89E-05	0.005288	0.003575	0.9971
SIGMA	0.040615	0.003771	10.77033	0

The application of the test to the remaining nine residual terms reveals that the all ten variables are all normally distributed with mean zero.

We proceed by testing for the serial correlation of residuals. We apply the Durbin Watson statistic test that yields a value of 2.085, showing that residuals are independent over time.

VARX is proved to be consistent and significant with normal independent residuals of zero mean. The model holds strongly with a high adjusted R^2 of 97.5%. The backward testing shows a significant similarity between the series generated by the estimated model and the actual data. This strengthens the performance of the model and its ability to predict future trends.

The following chart shows the estimated quarterly GDP figures plugged against the actual quarterly GDP-Chow Lin figures. The model reveals also very satisfactory when we computed the yearly difference in growth rates between the estimated yearly growth rates and the yearly growth rates:

Figure 3: *Estimated values in VARX vs. Actual Quarterly GDP figures (in billions of $US)*

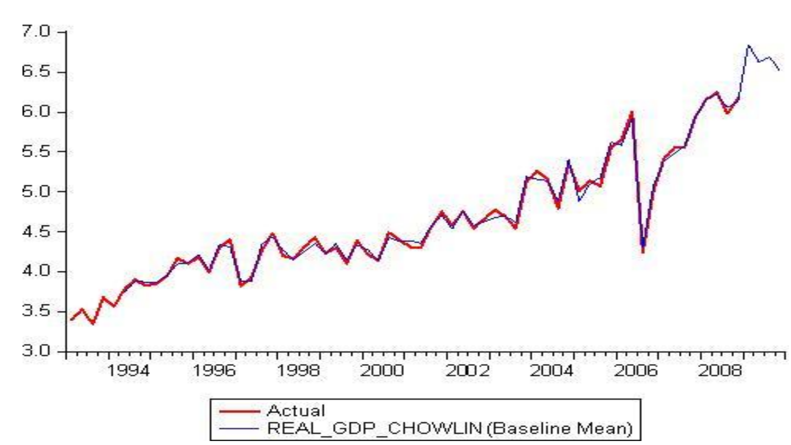


Table 5: *Estimated values in VARX vs. Actual Annual GDP figures*

	GDP estimated yearly growth	GDP actual yearly growth	Difference in Basis Points
1995	6.3%	6.5%	23
1996	5.1%	5.1%	3
1997	-2.0%	-2.3%	-29
1998	3.2%	3.6%	39
1999	-0.2%	-0.5%	-21
2000	1.2%	1.3%	17
2001	4.6%	4.0%	-59
2002	3.2%	3.4%	14
2003	3.5%	3.2%	-24
2004	7.6%	7.5%	-9
2005	1.0%	1.0%	-2
2006	0.5%	0.6%	5
2007	7.1%	7.5%	40
2008	9.3%	9.3%	-1
2009	8.6%	8.5%	-6

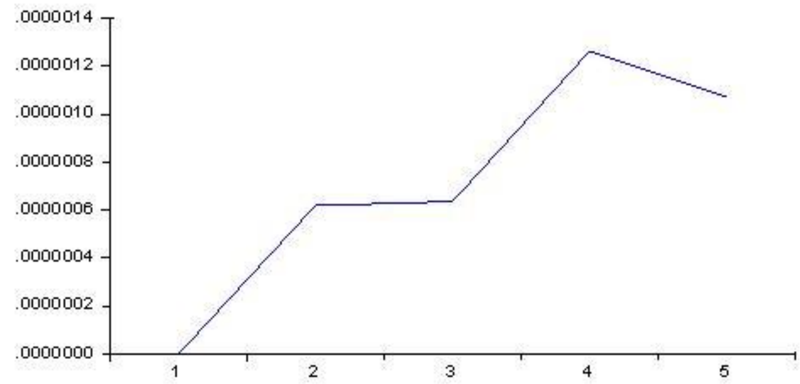
To obtain the estimated yearly growth rate of GDP for a given year, we add up the estimated values of GDP growth for the four quarters of the considered year. For 2009, real GDP growth is estimated at 8.6%.

VII- Impulse response

We generate impulse response functions for the VARX model to assess the sensitivity of estimation performance and certain policy analysis. We measure the impulse response of three endogenous variables on GDP: Claims on Private Sector, Passenger Flows, and Imports of Petroleum.

The following graph tracks the response of real GDP over time to a positive shock of one standard deviation on Claims on Private Sector. The shock on claims takes place in Quarter 1, keeping other variables fixed through the Choleski decomposition transformation:

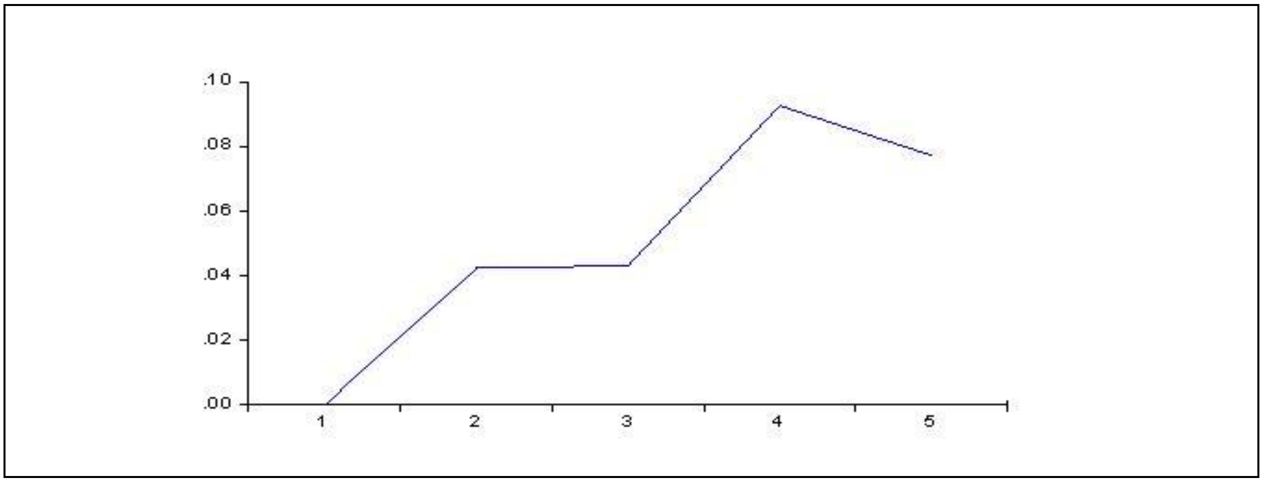
Figure 4: *Accumulated Impulse Response of Real GDP to a one Positive Standard Deviation Shock on Claims on Private* Sector



An external positive shock that would increase Claims on Private Sector, will impact real GDP positively over the course of a year (graphically, over four quarters, from 1 to 5). A flattening growth of GDP is observed in the second quarter (graphically, from 2 to 3). A plausible explanation is seasonality. The economic interpretation of the positive relationship between Claims on Private Sector and GDP is that an increase in the amount of loans extended by banks to the public - increasing by that the claims on private sector - will be targeted towards either households or investors. In the former case, consumers will use the loans to spend more on nondurable and durable goods. In the former case, businesses will make use of the loans to start up or expand their businesses. Thus, regardless of the targeted audience, a rise in private claims will impact positively GDP.

Another impulse response function worth observing is that of GDP to a one standard deviation positive change in Number of Arrivals at the Beirut International Airport. The Choleski decomposition yields the following:

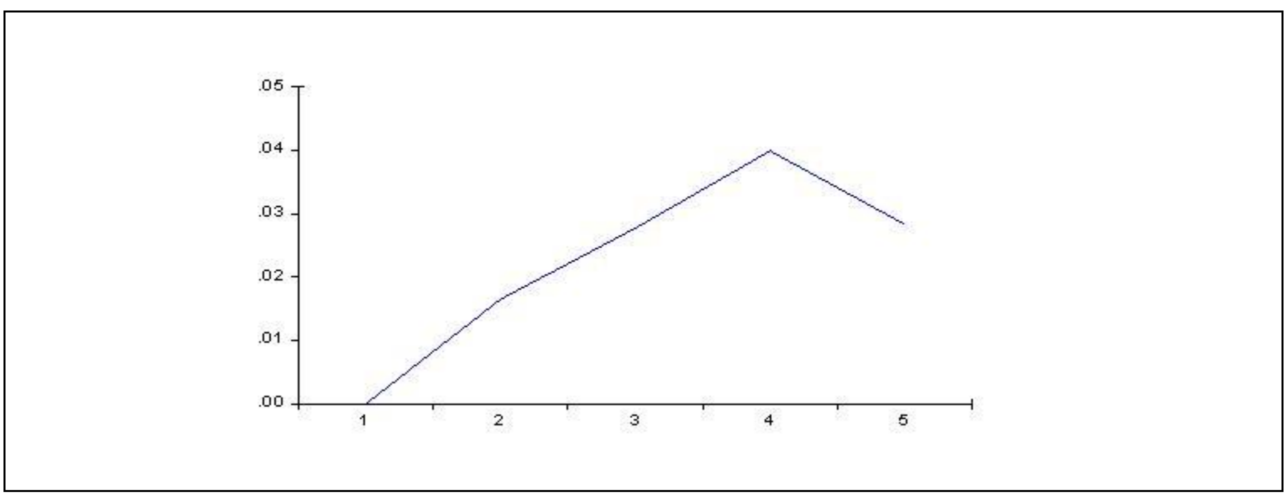
Figure 5: *Accumulated Impulse Response of Real GDP to a Positive Standard Deviation Shock on Arrivals*



A shock that would lead to a higher Number of Arrivals would positively impact real GDP over the next three quarters (graphically, from 1 to 5). The reason lies in the fact that more tourists would boost the level of spending lifting by that the level of economic growth.

We have also tracked the impulse response of GDP to a positive shock of a one standard deviation on Imports of Petroleum. The outcome entails the following:

Figure 6: *Accumulated Impulse Response of Real GDP to a Positive Standard Deviation Shock on Imports of Petroleum*



A rise in Imports of Petroleum, holding other variables constant, would cause an increase in real GDP over the course of a year (graphically, from 1 to 5). The upward trend in GDP will start to decelerate over the fourth quarter (graphically, from 4 to 5) due to the fading of the effect of the shock. That positive relationship between Imports of Petroleum and real GDP can be mostly reflected through the boost in consumption level, and partly through the rise in enterprises and manufacturing activities.

VIII- Conclusion

The VARX model developed in this paper indicates that when looking at historic GDP, the gap between the actual and fitted annual values lies in the range of -59bps; +40bps during the sample period extended from 1995 to 2008. This suggests a high level of accuracy and precision and validates the model as a useful tool in guiding estimation judgments'. The impulse response function conducted in the last section strengthens the estimation capabilities of the model as the results proposed capture the economic significance and are in line with the outcome of VARX. A more interesting and effective insight could be gained by making actual quarterly GDP growth available. If the Lebanese government intensifies its effort and develop further its National Accounts to have economic data, and more precisely GDP growth, published on a quarterly basis then we can assess better the economy and provide an even more accurate model. The precision of the estimated values would increase significantly in the latter context as the disaggregation stage would be omitted and thus, the margin of error in the estimation process would narrow.

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Appendix A - VAR Stationarity

1- Conditions for stationarity

The value of the scalar system of VAR (4) at time t is given by the following dynamic equation:

$$Y_t = c + \Phi_1 Y_{t-1} + \Phi_2 Y_{t-2} + \Phi_3 Y_{t-3} + \Phi_4 Y_{t-4} + \varepsilon_t$$

For our purpose it is more convenient to rewrite Y_t as a first order difference equation in a vector ξ_t. Define:

Vector ξ_t by $\xi_t = \begin{pmatrix} Y_t \\ Y_{t-1} \\ Y_{t-2} \\ Y_{t-3} \end{pmatrix}$

Matrix F for a 4th order by $F = \begin{pmatrix} \Phi_1 & \Phi_2 & \Phi_3 & \Phi_4 \\ I_{10} & 0 & 0 & 0 \\ 0 & I_{10} & 0 & 0 \\ 0 & 0 & I_{10} & 0 \end{pmatrix}$

And

Vector V_t by $V_t = \begin{pmatrix} \varepsilon_t \\ 0 \\ 0 \\ 0 \end{pmatrix}$

Under the above notation, consider the following first order difference equation which defines VAR (4):

$$\xi_t = F\xi_{t-1} + V_t$$

$$Where\ E(V_t V_\tau^{'}) = \begin{cases} Q & for\ t = \tau. \\ 0_{40\times40} & otherwise. \end{cases}$$

$$And\ Q_{(40\times40)} = \begin{pmatrix} \Omega & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{pmatrix}\ \Omega\ being\ the\ variance\ covariance\ matrix\ of\ \ \varepsilon_t$$

The effect of Y_{t+j} of a one unit increase in ε_t indicates whether VAR(4) is explosive or stationary: If the dynamic multiplier is between 0 and 1, then the absolute value of the effect decays geometrically towards zero and stationarity is satisfied. However, if the dynamic multiplier is lower or higher than 1, then the system would either exhibit explosive oscillation or the dynamic multiplier would increase explosively over time.

The analytical characterization of $\frac{\partial y_{t+j}}{\partial \varepsilon_t}$ is obtained in terms of the eigenvalues of the matrix F which are those numbers λ for which:

$$|F - \lambda I_{10}| = 0$$

The eigenvalues for VAR (4) are the solutions to:

$$|I_{10}\lambda^4 - \Phi_1\lambda^3 - \Phi_2\lambda^2 - \Phi_3\lambda - \Phi_4| = 0$$

Consequently, the VAR is stationary if the eigenvalues of the matrix F lie inside the unit circle.

2- Numerical values of the inverse roots

Root	**Modulus**	**Root**	**Modulus**
0.081447 + 0.924374i	0.927955	-0.001973 - 0.001973i	0.00279
0.081447 - 0.924374i	0.927955	-0.001973 + 0.001973i	0.00279
0.857306 + 0.214921i	0.883836	0.001973 - 0.001972i	0.00279
0.857306 - 0.214921i	0.883836	0.001973 + 0.001972i	0.00279
-0.847262	0.847262	-0.001915 + 0.000837i	0.00209
-0.532251 + 0.462054i	0.70483	-0.001915 - 0.000837i	0.00209
-0.532251 - 0.462054i	0.70483	0.000838 + 0.001915i	0.00209
-0.115952 + 0.634695i	0.6452	0.000838 - 0.001915i	0.00209
-0.115952 - 0.634695i	0.6452	-0.000837 + 0.001915i	0.002089
0.60437	0.60437	-0.000837 - 0.001915i	0.002089
0.430106 + 0.366042i	0.564782	0.001914 - 0.000837i	0.002089
0.430106 - 0.366042i	0.564782	0.001914 + 0.000837i	0.002089
-0.054897 + 0.423402i	0.426946	0.001809	0.001809
-0.054897 - 0.423402i	0.426946	-3.31e-06 + 0.001805i	0.001805
-0.332832 + 0.184468i	0.380533	-3.31e-06 - 0.001805i	0.001805
-0.332832 - 0.184468i	0.380533	-0.001802	0.001802
-0.007035 + 0.007141i	0.010024	-0.000890 + 0.000890i	0.001259
-0.007035 - 0.007141i	0.010024	-0.000890 - 0.000890i	0.001259
0.007035 + 0.006933i	0.009877	0.000890 + 0.000890i	0.001258
0.007035 - 0.006933i	0.009877	0.000890 - 0.000890i	0.001258

No roots lies outside the unit circle

VAR satisfies the stability condition

Appendix B - Vector Autoregressive Model (VAR)

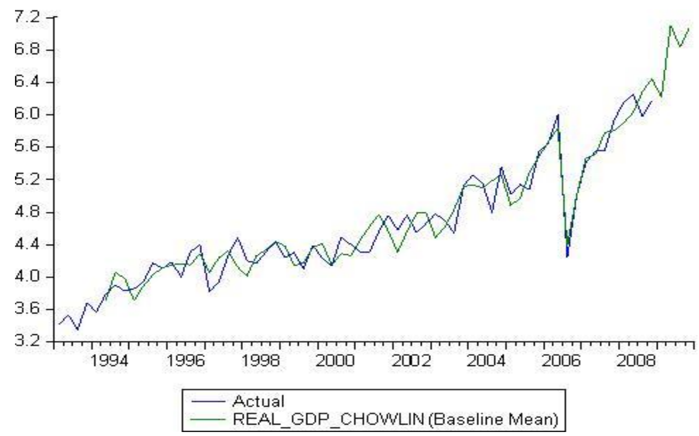
1- The model

Vector Autoregressive Estimates

	GDP	IP	L(C)	L(CP)	AR	L(GS)	L(TI)	TE	CPI	NRS
GDP(-1)	0.213881	-8.53E+04	-8.98E-03	0.122825	33974.27	-0.231274	-6.31E-02	1.26E+05	1.43E+00	1.52E+06
GDP(-2)	-6.83E-02	-2.62E+04	2.09E-02	2.22E-01	71259.63	-1.97E-01	-7.70E-02	4.77E+04	-2.75E+00	1.45E+06
GDP(-3)	-0.586348	106340.2	-5.81E-02	0.012843	4534.484	-0.109077	-2.79E-01	-82306.58	6.59E-01	-48072.83
GDP(-4)	0.639735	-8.62E+04	-2.63E-02	0.088467	20874.44	-0.095271	-2.85E-02	1.00E+05	-1.15E+00	4.40E+05
IP(-1)	2.00E-07	1.79E-01	-1.17E-08	-8.63E-09	0.038688	-5.65E-08	4.83E-08	-4.72E-02	7.56E-06	6.30E-01
IP(-2)	4.95E-07	4.45E-01	-9.71E-09	5.48E-08	0.074169	-1.82E-08	1.92E-07	-3.06E-02	9.09E-06	1.06E+00
IP(-3)	3.22E-07	3.17E-02	4.58E-08	-2.56E-08	0.035015	4.83E-07	9.99E-08	4.97E-02	5.29E-06	6.84E-01
IP(-4)	-1.10E-06	-4.00E-01	-3.57E-09	-1.47E-07	-0.083682	-7.06E-08	-4.09E-08	-2.36E-01	2.43E-06	-1.67E+00
L(C(-1))	-2.519215	-2.67E+06	1.49E+00	-1.147671	-115304	2.079967	5.21E-01	-2.05E+05	-7.12E+00	4.38E+06
L(C(-2))	5.22E+00	1.67E+06	-5.22E-01	2.45E+00	552813.2	-1.47E+00	-7.24E-02	6.40E+05	5.37E+01	1.31E+06
L(C(-3))	3.78E+00	3.94E+06	-2.66E-02	8.68E-01	908304.1	-1.18E+00	3.57E+00	-41034.68	-5.49E+01	9.01E+06
L(C(-4))	-5.65117	-2993809	0.070768	-1.883129	-1139715	0.87365	-3.127665	-261397	4.736882	-10091563
L(CP(-1))	0.2025	-30840.06	0.068221	0.320827	67963.53	0.328628	-0.530636	132021.7	13.46267	-1128596
L(CP(-2))	0.168992	181250.2	-0.085286	0.200527	-75452.9	-0.181593	0.318388	-36655.68	-5.918256	-947683.5
L(CP(-3))	0.47375	60940.52	0.061382	0.173079	100592.5	0.730947	0.072014	11776.54	-4.854846	2250200
L(CP(-4))	-1.15062	-380836.8	0.030197	-0.058805	-210598.9	-0.68176	0.13282	-54545.06	-3.894091	-2133070
AR(-1)	-2.19E-07	0.806437	1.01E-07	-1.29E-06	0.197097	1.57E-06	-1.29E-07	-0.765241	-3.80E-05	-6.349809
AR(-2)	-3.13E-06	-1.825251	1.54E-07	-1.22E-06	-1.10449	1.19E-06	-1.89E-06	-0.386387	2.32E-05	-19.21158
AR(-3)	6.86E-07	1.85918	-1.88E-07	-1.53E-06	-0.001856	-1.11E-06	1.20E-06	-0.674433	-1.85E-05	-9.214389
AR(-4)	3.05E-07	-0.746355	1.38E-07	-5.24E-07	-0.157053	2.17E-06	-1.30E-06	0.647581	-5.76E-06	0.426073
L(GS(-1))	0.027533	2.23E+05	-5.15E-03	0.004934	26008.35	0.589063	-3.35E-01	-5.51E+04	1.03E+00	5.13E+05
L(GS(-2))	-0.113608	-187094.7	-0.021964	0.122729	-41335.15	0.126369	-0.216284	-41005.06	2.528391	-1408977
L(GS(-3))	0.0069	-4.57E+04	-0.035888	0.108448	33893.58	-0.080328	0.233843	-65158.69	1.764728	95779.21
L(GS(-4))	-0.017177	116047.9	0.034349	-0.026194	-18344.77	-0.378437	0.389259	44286.42	5.41484	-280842.1
L(TI(-1))	0.085739	221583.7	-0.04238	0.204981	108320	-0.153024	0.516981	5154.507	-2.031739	1121131
L(TI(-2))	0.326528	-109351.9	-0.00229	-0.152156	-74735.54	0.138027	0.001569	19830.87	-4.865113	-1226677
L(TI(-3))	-0.509621	-164220	-0.014374	0.052666	-49878.24	-0.083156	-0.035629	-117448.1	5.26752	-760287.3
L(TI(-4))	-0.036373	183585.5	0.00192	-0.08248	-42295.48	0.12956	-0.000451	-23706.63	0.494241	-360848.6
TE(-1)	-3.34E-07	-0.829638	-1.07E-07	-7.04E-07	-0.560868	1.48E-06	-1.22E-06	-0.19455	2.88E-05	-8.764086
TE(-2)	5.76E-07	0.411937	-1.45E-07	-7.55E-07	0.432651	7.13E-07	1.73E-06	-0.06189	4.19E-05	2.383321
TE(-3)	1.83E-06	-1.452003	2.27E-07	2.07E-07	-0.1214	8.05E-07	8.49E-07	0.5526	3.68E-05	1.227829
TE(-4)	-6.85E-07	0.883946	2.05E-07	1.96E-07	0.56913	-1.61E-07	1.91E-06	-0.389103	2.40E-05	5.443346
CPI(-1)	0.011818	-3795.443	0.000447	0.003047	-742.067	-0.00154	-0.004419	2468.013	0.157105	23483.36
CPI(-2)	0.013789	13133.99	0.002028	0.00216	2504.592	-0.006653	0.008537	1054.766	-0.215236	46300.39
CPI(-3)	-0.016474	2226.156	-0.00124	-0.006276	-536.7748	0.006084	-0.012667	-5178.093	0.127589	-51086.16
CPI(-4)	0.002275	-170.0662	-0.000787	-0.00103	-679.0913	-0.003125	0.001939	3365.639	0.74305	-49614.66
NRS(-1)	-1.74E-09	-0.015551	-2.59E-09	-1.34E-08	-0.020622	-6.28E-08	3.49E-08	0.02804	-2.91E-08	0.387771
NRS(-2)	8.33E-08	0.115509	3.72E-09	1.48E-08	0.033348	-1.31E-08	8.26E-10	0.009548	-2.48E-06	0.812779
NRS(-3)	-2.72E-08	-0.089917	1.60E-08	6.24E-08	0.006904	5.43E-08	-5.80E-08	0.040944	-9.39E-07	0.198073
NRS(-4)	3.18E-08	0.004734	-1.77E-08	1.47E-08	-0.010638	-5.78E-08	2.76E-08	-0.032774	-4.51E-07	-0.11327
R-squared	0.941098	0.897052	0.999612	0.929356	0.948865	0.763847	0.922651	0.95527	0.99005	0.990748
Adj. R-squared	0.82624	0.696302	0.998856	0.7916	0.84915	0.303347	0.771821	0.868047	0.970647	0.972706
Sum sq.	1.634218	3.60E+11	0.001873	0.134976	3.00E+10	0.505761	0.347266	8.21E+10	185.0351	7.71E+12
S.E.	0.285851	134131.1	0.009678	0.082151	38747.26	0.159022	0.13177	64078.35	3.04167	620885.3
F-statistic	8.193558	4.468519	1322.344	6.746405	9.51584	1.658736	6.117141	10.95199	51.02652	54.91308

Where IP= Imports of Petroleum; L(C)= log of Claims on Private Sector; L(CP)= log of Cement Production; AR= Arrivals;
L(GS)= log of Government Spending; L(TI)= Total Imports Excluding Petroleum; TE = Total Exports; CPI= Consumer Price
NRS= Non-resdient Sepnding by Credit Card

2- Estimated values in VAR vs. Actual Quarterly GDP figures (in billions of $US)



3- Estimated values in VAR vs. Actual Annual GDP figures

	GDP estimated yearly growth	GDP actual yearly growth	Difference in basis points
1993			
1994		8.00%	
1995	2.48%	6.54%	405
1996	0.24%	5.14%	489
1997	-0.14%	-2.29%	-214
1998	4.45%	3.59%	-85
1999	1.87%	-0.45%	-233
2000	3.52%	1.34%	-217
2001	4.53%	3.95%	-57
2002	3.75%	3.37%	-38
2003	3.88%	3.24%	-65
2004	4.74%	7.48%	274
2005	1.81%	1.00%	-81
2006	5.41%	0.59%	-482
2007	9.59%	7.49%	-209
2008	6.05%	9.30%	325
2009	5.78%		

Appendix C - Residual Terms

1- Plot